Statistics of natural images as a function of dynamic range

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The statistics of real world images have been extensively investigated, but in virtually all cases using only low dynamic range image databases. The few studies that have considered high dynamic range (HDR) images have performed statistical analyses categorizing images as HDR according to their creation technique, and not to the actual dynamic range of the underlying scene. In this study we demonstrate, using a recent HDR dataset of natural images, that the statistics of the image as received at the camera sensor change dramatically with dynamic range, with particularly strong correlations with dynamic range being observed for the mean, standard deviation, skewness and kurtosis, while the one over frequency relationship for the power spectrum breaks down for images with a very high dynamic range, in practice making HDR images not scale invariant. Effects are also noted in the derivative statistics, the single pixel histograms and the Haar wavelet analysis. However, we also show that after some basic early transforms occurring within the eye (light scatter, non-linear photoreceptor response, center-surround modulation) the statistics of the resulting images become virtually independent from the dynamic range, which would allow them to be processed more efficiently by the human visual system.

Keywords: natural scenes, high dynamic range, statistics, retinal coding, spatial vision

Introduction

Attneave, 1954, was the first to apply principles of Shannon’s information theory (Shannon, 1948) to the human visual system (HVS) by demonstrating the predictability of visual information in images for human observers. In turn, Barlow, 1961, suggested that the role of early sensory neurons is to remove the statistical regularities present in the visual signal. Two examples of this efficient representation theory are the finding by Field, 1987 that the spatial frequency bandwidth and sampling of cortical cells are well matched to the spectral distribution of natural scenes such that the output of neighboring neurons will be decorrelated, while Hateren & Schaff, 1998, showed that properties of the independent component filters obtained with a large set of natural images match well with the behavior of simple cells. With regard to the visual modality, the identification of statistical regularities and the concept of redundancy reduction have proven critical to the fields of vision science, image processing and information coding (Simoncelli & Olshausen, 2001).

A large number of studies investigating the properties of natural images can be found in the literature, nearly all of which use regular, single-exposure photography. Dynamic range (DR) refers to the ratio of the maximum and minimum irradiance values in a scene or image; for instance, a uniformly illuminated scene may have a DR of less than two orders of magnitude, while a scene with direct sunlight and dark shadows may have a DR greater than...
seven orders of magnitude. In comparison, photography is limited by the DR of the camera sensor, which is typically less than three orders of magnitude, so a single picture of a high dynamic range (HDR) scene will suffer from the presence of underexposed and/or overexposed pixels. It was explicitly mentioned in the natural scenes databases of Schaaf & Hateren, 1996 and Geisler & Perry, 2011 that care was taken to avoid over and under exposed pixels resulting mainly in the capture of low dynamic range (LDR) scenes. Consistent with this, we estimate that the average DR of an image to be just 72 in the Schaaf and Hateren image database and 727 in the Geisler and Perry image database.

The aim of HDR photography is to capture the full range of light intensity values in any arbitrary scene, without suffering from overexposure or underexposure. A very popular strategy is to combine pictures of the same scene, taken with different exposure durations, into a single image (Debevec & Malik, 1997). The resulting HDR image is a spatially-varying weighted average of the single-exposure pictures. The weighting function is computed locally across the image and varies according the exact technique used, but will invariably ensure that overexposed and underexposed pixels receive a low weighting. Several studies have tackled the creation of HDR databases of natural images, e.g Fairchild, 2007; Xiao, DiCarlo, Catrysse, & Wandell, 2002; Parmar, Imai, Park, & Farrell, 2008; Adams et al., 2016.

To date, the topic of DR and image statistics has received only very limited attention. To the best of our knowledge, just two studies have compared the statistics of low dynamic range (LDR) and HDR images. The first, by Dror, Leung, Adelson, & Willsky, 2001, found that the properties of HDR images were similar in many respects to those of LDR scenes, except interestingly, that the well-established one over frequency statistic of the power spectrum of natural scenes breaks down for some images captured with HDR techniques, a point we shall return to later in this paper. The second, by Pouli, Cunningham, & Reinhard, 2010, reported a much higher skewness and kurtosis for the HDR intensity distributions with respect to the LDR ones.

A major drawback of both these studies is that they compared the statistics of images captured with multi-exposure HDR techniques with the statistics of single-exposure images, but did not consider how the statistics of the images vary with the underlying DR of the captured scenes. Therefore these works implicitly assume that an image in an HDR format, captured with a multi-exposure HDR technique, is “an HDR image”, while in fact this image might be representing an LDR scene.  

The contributions of this paper are, firstly, to evaluate how the statistics of natural images vary with the DR of the underlying scene, using for this purpose the recent Southampton-York Natural Scenes (SYNS) HDR image database (Adams et al., 2016), and secondly, to investigate how these statistics are affected by the early transformations of the visual system.

### Methods

The SYNS dataset includes scenes captured from both rural and urban locations in and around the city of Southampton in the United Kingdom. A total of 92 panoramic images in HDR format were recorded with a SpheroCam HDR using a Nikkor 16mm fisheye lens covering a range of 26 f-stops. Images are equirectangular 360° by 180° and each pixel represents an angle in space of 4 minutes of arc. The camera captures the full field of view by rotating about a point in space and uses multi-exposure capture and fusion so as to be able to record HDR content. Details about the database creation are provided in Adams et al., 2016, ISO, exposure time and aperture size that centered the median intensity value of each image are provided and these values were used to get the irradiance maps.

In order to enlarge the number of images we use for analysis, our database consists of crops of the SYNS dataset where we split each panoramic image into four 1348 × 1348 square images as illustrated in Figure 1. Each image corresponds to a 90° by 90° field of view. Two square images with zero values were discarded.

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1It is worth noting that the terminology in the literature is confusing, and the term dynamic range can refer to the capture technique, to the bit-depth of the resulting image, or to the DR of the resulting image.
from the database. The color spectra of the CCD sensors are available on the SYNNS website and this data was used to perform the calculation of the relative illuminance of each pixel. The transfer matrix was calculated with the values of the CIE Illuminant D65 obtained from cvrl.org and the reflectance spectra of 1269 Munsell color chips from uef.fi. Concerning the DR, it is computed as the contrast ratio (Ymax/Ymin) after the images are low pass filtered so as to reduce the impact of the noise as proposed by Mantiuk, Krawczyk, Myszkowski, & Seidel, 2007. The database was then divided into five different DR categories. The range was chosen such that each DR bracket contains either 72 or 73 images. The mean DR and range of each DR bracket are shown in Table 1.

<table>
<thead>
<tr>
<th>DR category</th>
<th>mean DR</th>
<th>min DR</th>
<th>max DR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>151</td>
<td>34</td>
<td>262</td>
</tr>
<tr>
<td>2</td>
<td>473</td>
<td>262</td>
<td>708</td>
</tr>
<tr>
<td>3</td>
<td>1053</td>
<td>708</td>
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<td>4562</td>
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<td>11200</td>
</tr>
<tr>
<td>5</td>
<td>342330</td>
<td>11200</td>
<td>6406500</td>
</tr>
</tbody>
</table>

Table 1: DR values for the different categories

Throughout this paper we investigate how the image statistics vary in each DR bracket, and also how these statistics change as they undergo three transformations corresponding to some very well-established processes taking place in the eye (this is clearly not intended to be a complete model of retinal processing).

The first of these transforms concerns light scatter due to the optics of the human eye and is modeled using the general glare equation provided in Vos, Berg, et al., 1999, as the filter:

\[ PSF(\theta) = \frac{10}{\theta^3} + \left( \frac{5}{\theta^2} + 0.1 \times \frac{p}{\theta} \right) \times (1 + \frac{a}{62.5}) + 2.5 \times 10^{-3} p \]  

where \( \theta \) corresponds to the viewing angle from the point from which the light is spread in degrees, \( a \) and \( p \) are parameters depending respectively on the age and the pigmentation of the eye of the subject. For this study, an age of 25 years and a value of 0.5 for brown eyes were chosen arbitrarily. The resulting point spread function (PSF) filter was then applied by convolution to the images of our database with the same method as in McCann & Vonikakis, 2016:

\[ R = \frac{I}{I + I_s} R_{\text{max}} \]  

where \( R \) is the response to the light stimulus, \( R_{\text{max}} \) the peak response, \( I \) the incident light and \( I_s \) is the semi-saturation level, i.e the intensity that causes half-maximum response; a global geometric average of the median and the mean was chosen for the computation of \( I_s \) as done in Ferradans, Bertalmio, Provenzi, & Caselles, 2011.

Finally, to simulate the impact of center-surround modulation on the spatial frequency profile of neurons such as retinal ganglion cells, we use the model given by Enroth-Cugell & Robson, 1966 that defines this filter:

\[ CSF(f) = K\pi (k_c r_c^2 e^{\pi r_c f} - k_s r_s^2 e^{\pi r_s f}) \]  

where \( f \) corresponds to the frequency in cycles/degree, \( r_c \) and \( r_s \) to the radii of the center and surround areas, and \( K, k_c \) and \( k_s \) the parameters of the filter. \( r_c, r_s \) and the ratio \( k_s r_s^2 / k_c r_c^2 \) are taken from the first line of the table given in Enroth-Cugell & Robson, 1966, \( k_c \) and \( K \) are respectively chosen as 1 and 10 to get transformed values in the same order of magnitude as the ones before applying \( CSF \).

To summarize, each input image from the database is considered as the real-world illumination reaching the eye. This signal is convolved with the PSF of Equation 1 simulating the optics of the human eye. Next, this new image is passed through the Naka-Rushton Equation 2, putatively modeling the response function of photoreceptors. Finally, we pass the image resulting from the previous step through the filter of Equation 3 to emulate the impact of lateral inhibition on contrast sensitivity.

For the statistical analysis, we choose to perform studies on single-pixel, derivative and wavelet statistics with the exact same methodology used in Huang & Mumford, 1999, which is a very well-established work on natural image statistics based on the LDR database of Schaaf & Hateren, 1996. This will allow for a better comparison of the new results we obtain here for HDR images, and also to check the validity of our study since the statistics for the lower DR category of the database we are using should be consistent with those reported in Huang & Mumford, 1999.
Results

Statistical moments

We begin by investigating how the statistical moments of mean, standard deviation, skewness and kurtosis vary as a function of DR. The results are presented in Figure 2; each subplot reports a separate moment, each dot represents an individual image and each color denotes a different transform, with blue for the original images, red for the results of the Naka-Rushton transform and yellow for the result of the center-surround modulation. For clarity we have omitted the plots for optical scatter as the transform (Equation 1) was deemed to have a very small impact upon the statistical moments. Each image is normalized (divided by its maximum) prior to computing the moments. For the skewness, some negative values have been found and marked with a black cross on their absolute values plot in log axis.

Figure 2: Statistical moments of intensity distributions as a function of DR: mean (A), standard deviation (B), skewness (C), kurtosis (D).

For the first moment, Figure 2(A), we observe a strong relationship between the mean and DR. This effect is so strong that for images with a DR greater than $10^5$ over 95% of pixel values are found within the first 1% of the illuminance range. After the Naka-Rushton equation is applied, the effect is substantially reduced in strength and it remains like this after the CSF transform.

In Figure 2(B) we can see that for the original images the standard deviation decreases with DR (recall that we first normalize the image between zero and one, otherwise standard deviation increases with DR). As was the case for the mean, application of the Naka-Rushton equation strongly reduces this correlation and results in the standard deviation hovering around the 0.2 mark irrespective of the DR: by comparison we would expect a uniform distribution in the range $[0, 1]$ to have a standard deviation of 0.29. Again the CSF filter keeps low the correlation with DR and simply reduces the magnitude of the second moment.

In Figure 2(C) we can see that LDR natural scenes are typically positively skewed (concentrated towards low values) and this effect increases strongly with the DR. After the Naka-Rushton equation is applied, the skewness no longer increases with DR. And after the CSF filter, skewness values are much less spread.

A similar phenomenon can be observed in Figure 2(D) for kurtosis. We note that kurtosis values are 3.0 for a normal distribution and 1.8 for a uniform distribution, and the figure shows that for LDR images the kurtosis values are relatively close to those numbers, but then they increase very rapidly. Again the impact of DR on kurtosis is greatly reduced by the application of the Naka-Rushton equation, and the effect is maintained after the CSF transform.

The main observation is that the linear images, represented by the blue dots, have statistical moments that vary strongly with DR for each of the statistical moments evaluated, but that this effect size is greatly reduced by the application of the Naka-Rushton equation, as depicted by the red dots. Overall, we can conclude that the application of the Naka-Rushton greatly reduces the impact of DR on the statistical moments of the illuminance distributions.

Single pixel statistics

In the previous section we evaluated the image statistics as a function of their DR on an image-by-image basis. In this section we shall examine the average shape of the histogram for each of the five DR brackets described in the methods section (Table 1). To do so, we use the methodology of Huang & Mumford, 1999 in which each image is passed through a log transform before the median is subtracted, as follows, $H = \log(I(i, j)) - \text{median} (\log(I))$. Average histograms are then computed for each DR bracket and the results are plotted on linear-log axes. In Figure 3 we plot the average histograms of the original linear image in subplot A and remaining transforms in subplots B, C and D. Each color represents a different DR category.

The width of the histograms of the original linear images un-
surprisingly increases with DR. Interestingly, we only observe straight edges in the histogram for the of highest DR bracket we evaluate. This contrasts with the work of Huang & Mumford, 1999, in which straight edges are found for the average histogram of low DR database of Schaaf & Hateren, 1996. We speculate upon why this is the case in the discussion where we replicate the original findings.

Figure 3: Median-subtracted histograms in log-log axes, by DR categories. (A) For the original values. (B) After light scatter, Equation 1. (C) After photoreceptor response, Equation 2. (D) After center-surround modulation, Equation 3.

Regarding the impact of the eye’s PSF on illuminance distributions, represented in Figure 3(B), one can observe that the impact is limited to raising the low illuminance values while having very little impact upon values above the median. The PSF models imperfections in the eye’s medium that cause light scatter, and whose net effect is to increase the minimum intensity level of the light reaching the retina photoreceptors (Stiehl, McCann, & Savoy, 1983).

Figure 3(C) shows that after passing through the non-linear transform of the photoreceptors, Equation 2, the average histograms for all DR categories become much more similar and also their DR is greatly reduced (the width of the histograms is much smaller).

Finally, in Figure 3(D) we can see that application of the filter of Equation 3 produces images where the similarity of histograms across DR brackets is preserved, even enhanced, but now the DR has been notably increased.

Figure 4: Histograms of the difference between adjacent pixels in linear-log axis. (A) For the original values. (B) After light scatter, Equation 1. (C) After photoreceptor response, Equation 2. (D) After center-surround modulation, Equation 3.

**Derivative statistics**

Derivative statistics correspond to the difference between two adjacent pixels. We find similar histograms for both horizontal and vertical derivative statistics, and as such we only plot results for the horizontal derivative distributions (computed with the formula $D = \log(I(i,j)) - \log(I(i,j + 1))$), represented in Figure 4.

For the original images, Figure 4(A), we see that as the DR increases the adjacent pixels typically become more different and the width of the difference histograms widens.

Light scattering has the effect of making these distributions more similar for all categories and also closer to zero values, see Figure 4(B). This result was expected since the PSF filtering affects neighboring pixels and reduces differences between them.

The distributions for the five DR brackets become even more similar and their spread is reduced even further after applying the Naka-Rushton transform, see Figure 4(C).

Finally, the contrast enhancement effect of the center-surround modulation provided by filtering with Equation 3 can be clearly seen in Figure 4(D), where the distributions remain very similar for all DR categories but the spread is now greatly increased.

**Power spectra**

The literature about natural image statistics has consistently demonstrated that the power spectra of natural scenes is linear
when plotted on log-log axes. This corresponds to a $1/f^{2+\eta}$ power law relationship, with $\eta$ called the “anomalous exponent”, usually small. An implication of this feature is that natural images are scale-invariant (Field, 1987; Burton & Moorhead, 1987; Tolhurst, Tadmor, & Chao, 1992; Ruderman & Bialek, 1994; Schaaf & Hateren, 1996).

In the previous works of Dror et al., 2001 and Pouli et al., 2010, it was found that this rule was not valid for some HDR images. They associated this finding with the presence of direct light sources and/or specularities in the image. In this study, we investigate the effect of the DR on the averaged power spectrum of each image after collapsing across orientation.

Figure 5(A) shows the mean of power spectra for the five DR brackets. Substantial curvature is observed for the highest DR category of the dataset, corresponding to a flattening of the distribution at low spatial frequencies. Some flattening may also be seen in the second-highest DR category. The effect of the eye’s PSF, Figure 5(B), results in a small reduction in the amplitude of the high spatial frequencies. After application of the Naka-Rushton equation we can see in Figure 5(C) that the $1/f^2$ relationship is recovered. Finally, Figure 5(D) depicts the result of the center-surround transform, which decreases the energy in the lower frequencies and also in the frequencies over 100 cycles/image. The CSF acts like a bandpass filter resulting in the effect observed in Figure 5(D).

We wanted to focus on the recovery of the scale-invariance property of natural scenes. As mentioned in Dror et al., 2001, the curvature present in the power spectrum of HDR images can be due to strong localized light sources producing very high illuminance values over small regions. To study the image-per-image evolution of this statistic, a second order polynomial was fit to the averaged power spectrum of each natural image, $P(x) = ax^2 + bx + c$, where $a = 0$ would correspond to the scale-invariant case where the power spectrum can be approximated by a linear function, and $a \neq 0$ implies some curvature in the fit. These fits were performed up to 200 cycles/image to avoid an overestimation of the fitting error that would be produced by the drop on the high frequencies due to the PSF.

Figure 6 plots the term $a$ as a function of DR, where negative values of $a$ correspond to an inverted U-shape. The blue dots show fits to the original images, and it can be seen that the coefficient $a$ becomes more negative with increasing DR. The red dots show the fits to the data after the image has been passed through the eye’s PSF and the Naka-Rushton equation. As the figure shows, the fits do not exhibit the same degree of negative curvature at high DR.
values. In Figure 7(A) we plot the fitting error for a first and second order polynomial fit respectively in blue and orange for the camera sensor output, and in Figure 7(B) we plot the error for the images after modeling the effect of the light scatter and the photoreceptor response. The main observation that can be done is that the curvature of the power spectra is reduced after the application of Naka-Rushton equation. Indeed, in Figure 6 the absolute value of the $a$ coefficients are below 0.1. And, although Figure 7(A) shows that even a second-order fit may have significant error, the application of the Naka-Rushton equation allows for the power spectra to be well approximated by a first-order polynomial, as the small errors of Figure 7(B) suggest.

In Figure 6, one can see that the curvature is evolving continuously as a function of the DR for the original values. HDR scenes tend to have a curved power spectrum, and this curvature can be due to high specularities as reported before (Dror et al., 2001), but it is not systematical. For instance, in Figure 6 some images with a high DR have a value of $a$ close to zero or a small error of the first order polynomial fit in Figure 7(A). These results can highlight the fact that some HDR images comply with the 1/f rule.

Wavelets

Following Huang & Mumford, 1999 we study joint statistics in the wavelet domain, deconstructing the image into subbands of different orientations and spatial scales. For this purpose we perform a Haar wavelet analysis of the database (after removing a few images suffering from motion artifacts or smear). Joint histograms are shown in Figures 11, 12 and 13 as contour plot of log(histograms) of 8 wavelet coefficients pairs. The finest possible scale was used for the joint distributions, so “horizontal component”, “vertical component” and “diagonal component” refer to the wavelet coefficients of the first subband for each orientation. According to the nomenclature used in Huang & Mumford, 1999, these components are cousins to one another, coefficients at adjacent spatial locations in the same subband are called brothers, and coefficients that correspond to the same spatial location at different scales are called parent and child.

Figure 11 plots joint wavelet histograms computed on the original images for the lower DR category (top panel) and the higher DR category (bottom panel). One can observe differences among categories for all the corresponding joint histograms, especially sharper vertices for the histograms of higher DR images. The higher DR category contains also histograms with lower variance, values of the Haar wavelet coefficients are closer to zero.

Figure 12 plots joint wavelet histograms computed on the images after applying Naka-Rushton equation (the stage with the PSF is omitted because of its small effect on the statistics). After photoreceptor transduction the wavelet statistics have more similar variance but some shape differences remain like the sharpness of the vertices. The last transform, the center-surround modulation, whose wavelet statistics are represented in Figure 13, makes the joint histograms even more similar between the two extreme DR categories. The shapes are also changed significantly.

Discussion

The results reported above show that the statistical analysis of image databases that include mid to high DR images will lead to dramatically different results depending on the DR of the image in question.

But we have observed that these differences disappear or are substantially weaker after we pass the images through some of the transformations that occur in retinal processing. For instance, the first to fourth statistical moments vary substantially less with DR, the average histograms become more similar across the DR brackets evaluated, and the curvature seen in the power spectra of the highest DR category disappears after a non-linear transform representing a process as early as photoreceptor transduction. Thereby, past studies on natural image statistics addressing later processes in the visual system are not invalidated, because images with a high DR will have the same properties as LDR images after being absorbed by the photoreceptors. Furthermore, if one assumes that the HVS is adapted for natural scenes as studied before with single exposure images, this property can extend to HDR images regarding the similarity of the intensity distributions after the application of the transforms used in this paper.

Assumptions regarding the nature of HDR scene illumination distributions can also be made with these results. When observing the evolution of the moments in Figure 2, one can see that illumination maps go to highly positively skewed and leptokurtic distributions when the DR increases, that could correspond to images with highlights. The curvature of the power spectra of most of the HDR images can also support the hypothesis that HDR scenes are characterized by specularities and bright light sources, that flatten the low frequency part of the power spectrum as pointed by Dror et al.,
Although, it is important to notice that some HDR images do not present a curved spectrum. In Figure 6 they correspond to blue dots in the top-right corner, with a high $\alpha$ and a high DR. An example of this image with its power spectrum is presented in Figure 8. In order to be displayed properly the image is tone mapped so one can’t see the specularities in the car and the truck on the background. This scene may abide by the $1/f$ law because of the multiple specularities and their distribution in space, opposed to a local light source that would indeed flatten the power spectrum.

Some HDR images can have also a more complex shape than this flattening in the low frequencies. Most images have a power spectrum well represented by a second order polynomial but some exceptions with a relatively high fitting error are noticed in 7(A). We show one example of this complex power spectrum in Figure 9. Here, the sun is present in the top-left corner of the image forming a local light source. The geometry of the scene can affect the power spectrum and make it take a shape that can only be approximated with a third order polynomial. No further study on this topic is presented here but one can agree on the fact that specular highlights affect the power spectrum of natural scenes as it was stated in a previous study (Dror et al., 2001).

Another interesting feature is the increase of the DR of the signal after lateral inhibition observed in Figure 3(D). The first stages of visual processing performed by the HVS are way more complex than transforms described in this study. Many different types of ganglion cells exist with functions and size varying substantially (Sanes & Masland, 2015) and one can argue that strict spatial information is incomplete when studying the HVS and still images do not represent the fundamental spatiotemporal information reaching the eye (Rucci & Victor, 2015). Still, with basic models of processes occurring in the retina one can have a hint of the functions of the different stages. Regarding Figure 4(D), a wider derivative histogram after applying the center-surround modulation is in agreement with the decorrelation hypothesis developed by Atick & Redlich, 1992. On the other hand, the power spectra in Figure 5(D) are flattened for the frequencies below 100 cycles/image and support the response equalization hypothesis of Field, 1987. A debate including both theories is proposed in Graham, Chandler, & Field, 2006, but they agree on the fact that the retina aims at “whitening” the visual input which is in accordance with the effects observed after this last transform.
image database, showing that it was well approximated in piece-wise linear formulation when plotted on log-log axes. There was however, some ambiguity as to how the original histogram was computed, as Huang & Mumford, 1999 stated only that images underwent the following transformation: $\log(I) - \text{average}(\log(I))$. Thus it was not clear whether average referred to the mean, the median, or some other computation. Additionally, the Schaaf and Hateren image database contains two image sets denoted .iml and .imc, the former being images that are linearly related to the sensor values and the latter having a correction for the optics of the camera applied. In Figure 10, we test the four possible combinations using the Schaaf and Hateren image database. The results demonstrate that we only obtain the characteristic linear slopes when we use the .imc, optically corrected image dataset and subtract each image using the image median. As such, we use the median to compute the histograms in Figure 3. The gray dashed box highlights the region illustrated in the original study by Huang and Mumford which is substantially smaller than the region we plot here, even if one can argue that these parts of the histograms contain a very small amount of pixels. When plotted over this greater range we do not obtain a straight line over the full range of positive values. In the SYNS dataset, images are not optically corrected and it may be a reason why we do not observe linear parts on the histograms of small DR categories. It is to be noticed that the number of images per category is way smaller in this study than in the previous one, we have around 70 images for each category while in Huang & Mumford, 1999 the dataset contains more than 4000 natural scenes. The type of scenes can then affect the histograms, for instance in Figure 3(A) one can observe a peak in the positive part of the histograms. This peak may correspond to the pixels forming the sky as the SYNS dataset contains a lot of scenes with the horizon creating binary intensity distributions.

However, a piece-wise linear histogram is observed for the HDR category and one explanation would be that HDR images are less affected by optical scatter regarding the distribution they have. It is hard to make a strong conclusion about the real world illuminations given the limitations imposed by the capture devices. One solution is to use physically-based computer graphics methods which bypass the need for an optical system. Additionally, one can gain complete access to the reflectance and illuminance distributions, potentially allowing us to develop a deep understanding of why natural scenes have the illuminance distributions they do.

### Conclusion

This study examines the impact of dynamic range on the statistics of real world images. The vast majority of studies into image statistics use single-exposure photography, which is constrained to less than three orders of DR without substantial over or under exposure artifacts. HDR techniques, on the other hand, allow the capture of scenes with a much wider DR, including images with direct light sources such as the sun or artificial lighting, which can often have an extremely high DR ($> 10^7$), but also scenes with deep shadows and high contrast which span the mid-dynamic ranges.

The results in this paper demonstrate that the statistics of the linear image, that is, the image received at the camera sensor, change dramatically with DR. The strongest effects are noted in the four statistical moments evaluated here and upon the one over frequency relationship for the power spectrum which breaks down for images with a very high DR. Effects are also noted in the derivative statistics, the single pixel histograms and the Haar wavelet analysis.

The second conclusion from this paper is that the early transformations of the human visual system, and in particular the Naka-Rushton non-linearity that models photoreceptor responses, greatly reduce the impact of DR on the statistics, turning the four statistical moments virtually independent from DR and recovering the $1/f^2$ behavior for the power spectrum.

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Figure 11: Wavelet analysis of original values for the lowest DR category (< 262) (A) and for the highest DR category (> 11200) (B).

Figure 12: Wavelet analysis after NR for the lowest DR category (< 262) (A) and for the highest DR category (> 11200) (B).
Figure 13: Wavelet analysis after CSF for the lowest DR category (A) and for the highest DR category (B).